Stem Taper and Volume of Managed Red Alder

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ABSTRACT

A taper equation and a volume table are presented for red alder (*Alnus rubra* Bong.) trees grown in plantations. Fourteen diameter measurements from each of 234 trees were collected from nine plantations throughout the Pacific Northwest. Diameter inside bark (dib) along the stem was fitted to a variable exponent model form. Individual tree merchantable volume was then estimated as volume inside bark by integrating the taper function from 6 in. (stump height) to the height at a 5-in. (diameter outside bark) top. Incorporating two easily measured tree variables— dbh and total tree height—provided an accurate fit. Model results and the use of an independent evaluation data set of plantation-grown trees indicated that the model presented here was a better predictor of dib in managed stands than previously published red alder taper equations. This equation provides reliable dib and merchantable volume predictions and is an improvement over previous red alder volume and taper equations.

Keywords: red alder, taper equation, volume tables, stem form

The forests of the Pacific Northwest have contained large volumes of red alder (*Alnus rubra* Bong.) as a result of both the historic disturbances by fire and landslide and the logging disturbances in the first half of the 20th century (Little 1978, Hibbs et al. 1994, Raettig et al. 1995). Over the last 30 years, this large volume of a very useful wood has led to the development of a large hardwood processing industry. The gradually increasing value of this industry and of red alder wood has led to an increased interest in management of red alder in plantations (Deal and Harrington 2006).

Through the development of this industry, tools have been devised to evaluate the volume of individual red alder stems and for stands scheduled for harvest. These volume estimates have been of two types: volume equations (Johnson et al. 1949, Skinner 1959, Browne 1962, Snell and Little 1983) and taper equations (Curtis et al. 1968, Kozak et al. 1969, Kozak 1988, 1997, 2004). All these volume and taper equations are based on stands of natural origin, absent of any management activity.

Stand conditions and management activities affect tree diameter and in some cases, height, which can affect, in turn, stem shape and stem volume (Larson 1965, Hilt and Dale 1979, Valenti and Cao 1986, Lennette 1999, Garber and Maguire 2003). For red alder, the differences in tree growth between natural and planted stands (DeBell and Harrington 2002) may result in differences in stem shape as well, leading to an unknown amount of error in volume estimation. With the current interest in managed red alder, the decrease in merchantability limits, and the increase in unit value of red alder, a new tool to account for the effects of management and to more accurately assess tree volume is needed.

Taper equations are just the tool. They can be used to estimate diameter inside bark (dib) anywhere along the stem, inside bark volume of the entire stem, to any top height diameter and from any stump height, and between any two points along the stem (i.e., individual log volumes). Thus, the objective of this project was to develop a taper equation and a merchantable volume table specific to plantation-grown red alder. Stem profile (i.e., dib) was modeled with a variable exponent model (Kozak 1988, 1997, Newnham 1992, Kozak and Smith 1993) and volume was then estimated by integrating the taper function. The final model and associated volume table use dbh and total tree height (hereafter abbreviated as ht) to estimate individual merchantable tree volume (5-in. top diameter outer bark [dob], 6-in. stump, and minimum log length 10 ft). The volume table presented here is an effective tool that yields better volume estimates than previous tools when applied to the plantation-grown red alder.

Methods

The primary data set in this analysis came from 234 trees in nine plantations of pure red alder in western Oregon, western Washington, and southwestern British Columbia. The oldest plantations (i.e., the biggest trees) across a wide geographic range were selected. Trees ranged in dbh from 3.7 to 10.7 in., in height from 30.2 to 77.8 ft, and in age from 11 to 15 years. Mean site index (Harrington 1986) was 104 ft (base age, 50 years) and ranged from 90 to 115 ft (Table 1). At each site, inoculated, local red alder nursery stock were planted in randomly assigned blocks and within each planting block, control plots and thinning and pruning treatment plots were randomly assigned also. Treatment activities and data collection are administered by the Hardwood Silviculture Cooperative, Department of Forest Science, Oregon State University, Corvallis, Oregon (see OSU CoF 2005 for more details of this regional silviculture study).

Three types of plots were sampled at each site. First, three control (unthinned and unpruned) plots were sampled: 230, 525, and

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Table 1. Site and tree characteristics for the modeling data by site.

Site name	Pollard	Pioneer	Siletz	Sitkum	Shamu	Humphrey	Clear Lake	Mohun	Thompson
Latitude (deg, min)	45.20	44.62	44.73	43.13	45.93	48.58	48.47	50.67	45.48
Longitude (deg, min)	123.85	123.87	123.88	123.87	122.20	122.18	123.03	122.30	123.78
Elevation (ft)	1,100	350	300	1,000	1,100	400	500	325	1,225
Slope (%)	10	10	5	12	12	5	35	15	7
SI (50; ft)	100	110	115	110	95	115	105	100	90
Estimated date	1991	1992	1994	1992	1992	1989	1990	1993	1992
Sample size									
Trees	30	26	22	29	30	20	19	28	30
Stem diameters	420	364	308	406	420	280	266	392	420
Stem dob (in.)									
Minimum	0.2	0.3	0.2	0.3	0.2	0.3	0.3	0.2	0.2
Mean	4.1	4.5	3.7	4.7	4.4	4.8	4.6	3.6	4.1
Maximum	10.5	13.6	9.3	13.7	12.3	12.3	11.4	9.1	10.3
Tree dbh (in.)									
Minimum	3.7	5.1	4.3	3.8	4.1	3.9	4.6	3.9	3.7
Mean	6.7	6.8	5.8	7.1	6.6	7.3	7.2	5.5	6.3
Maximum	9.6	10.4	7.8	10.7	10.0	10.9	10.7	7.0	8.9
Tree height (ft)									
Minimum	32.3	40.7	30.2	51.2	33.8	45.6	49.2	32.8	30.4
Mean	45.6	52.5	40.4	59.1	50.2	62.0	56.4	43.6	46.6
Maximum	58.1	59.9	47.2	66.9	63.7	77.8	63.7	57.4	55.5

SI, Site index is the mean height of 40 dominant and codominant trees per acre (50-year base) calculated from Harrington 1986.

1,200 trees per acre (tpa). Second, a 230-tpa pruned plot was sampled. All pruned plots have had three 6-ft lifts. The first pruning lift occurred when tree heights were approximately 15 ft (between the ages of 4 and 6 years). The third lift was performed in the dormant season just before sampling on the majority of plots. Third, various thinned plots were chosen with special emphasis on plots planted at 525 tpa and thinned to 230 tpa between ages the 6 and 12 years (i.e., late thin), because this treatment is most representative of operational management. If this treatment was not present on a site, then another thinning treatment was sampled, either a plot planted at 525 or 1,200 tpa and thinned to 230 tpa at the age of 5 years (i.e., early thin). Diameter distributions were generated from the most recent data and sample trees were selected across the full range of diameters found in each plot. See Table 2 for the distribution of sample trees by site and treatment. At each plot, one to three (generally two) trees of good form (no forking, broken tops, excessive sweep, and more) from each 2-in. dbh class were selected at random. The sampling procedure used here was the one used for the Inland Northwest Growth and Yield Cooperative Tree Form Equation Project (Hatch and Flewelling 1995). The dob and double bark thickness (dbt) were measured directly at 14 points along each stem: at dbh (4.5 ft), three locations below dbh, and every 10% of height above dbh. The dib was calculated as dob − dbt.

After model fitting, an independent data set was used to evaluate the model dib predictions. Volume prediction was not assessed using this data set because of a limited number of dib observations per tree. The data set was 40 trees (2.5- to 9.4-in. dbh) from three managed stands in western Oregon and Washington (D. E. Hibbs, unpublished data). See Table 3 for evaluation data set site and tree characteristics.

Several equation forms were screened for modeling stem shape, including several segmented polynomials (Max and Burkhart 1976, Walters and Hann 1986), trigonometric forms (Thomas and Parresol 1991), a model presented by Zakrzewski (1999), and variable exponent (Kozak 1988, Bi and Long 2001). The variable exponent model showed the smallest average bias along the stem and therefore was chosen as the basis for the final model:

$$\text{dib} = a_1 \text{dbh}^{a2} X^{C} + \varepsilon \qquad (1)$$

where $X = [1 - (Z)^{0.5}]/[1 - (P)^{0.5}]$; Z is the relative height h/ht; h is the height above the ground (ft); $P = 4.5/\text{ht}$; C is a function of Z; dbh/ht, a_1, and a_2 are the parameters estimated from the data; ε is $N(0, \mathbf{\Sigma})$, and the other variables are defined previously. In this case, dbh was chosen as the reference point so that $X = 1$ at dbh and therefore the dib prediction was constrained to equal the bark factor equation at this point.

Table 2. Red alder sample distribution, by site and treatment. numbers in parentheses are years since thinning.

	Treatment							
Site name	230 tpa Control	230 tpa Prune	525 tpa Control	525 tpa Early thin	525 tpa Late thin	1200 tpa Control	1,200 tpa Early thin	Total
Pollard	6	6	6	–	7 (1)	4	–	29
Pioneer	6	4	6	–	6 (4)	4	–	26
Siletz	6	–	4	4 (5)	–	4	4 (5)	22
Sitkum	7	6	6	–	8 (3)	3	–	30
Shamu	6	6	7	–	6 (4)	5	–	30
Humphrey	8	–	8	–	8 (9)	6	–	30
Clear Lake	7	–	6	–	–	6	–	19
Mohun	4	4	4	4 (7)	–	4	–	20
Thompson	6	6	6	–	6 (4)	4	–	28
Total	56	32	53	8	41	40	4	234

Table 3. Site and tree characteristics for the independent evaluation data.

Site name	Apiary	Centralia	Cascade Head
Stand type	Planted	Planted	Planted
Location	Rainier, OR	Olympia, WA	Lincoln City, OR
Elevation (ft)	980–1,150	250–450	1,082
Slope (%)	5–20	NA	0–10
Age when sampled	20	13	9
Range of densities (tpa)	540–5,476	112–3,923	67–440
Sample size			
Trees	9	14	17
Stem diameters	87	89	81
Stem dob (in.)			
Minimum	0.2	0.2	1.1
Mean	4.3	3.5	9.4
Maximum	11.0	10.4	4.2
Tree dbh (in.)			
Minimum	5.0	2.5	4.1
Mean	6.4	5.3	8.5
Maximum	9.4	7.8	5.6
Tree height (ft)			
Minimum	57.9	25.4	27.3
Mean	66.5	37.3	33.4
Maximum	73.2	53.6	43.2

Several existing parameterizations (Kozak 1988, 1997, 2004, Garber and Maguire 2003) for the variable exponent C were tested. However, most of these parameterizations resulted in excessive under- or overestimation of dib below breast height. Therefore, an alternative approach was taken, reparameterizing C as two additive nonlinear functions of Z, dbh, and ht representing the sections below and above breast height:[1]

$$\mathrm{dib} = a_1 \mathrm{dbh}^{a_2} X^{a_3[1.364409 \cdot \mathrm{dbh}^{1/3} \cdot \exp(a_4 * Z) + \exp(a_5(\mathrm{dbb/ht})^{a_6} \cdot Z)]} \qquad (2)$$

To facilitate valid statistical tests for model identification, heterogeneous variance and spatial autocorrelation needed to be accounted for. Preliminary model evaluation suggested an equivalent reduction in the impact of autocorrelation on statistical tests but better dib and volume prediction with generalized least squares as opposed to mixed effects. Equation 2 was then fit to the data in nonlinear form, specifying $\boldsymbol{\Sigma}$ as a block diagonal matrix by incorporating a power variance function. A first-order continuous autoregressive process on Z by tree was used to account for heterogeneous variance and autocorrelation, respectively, using maximum likelihood. Evaluation of assumptions for testing parameters was assessed with residual and empirical autocorrelation plots and parameter estimates, variance functions, and correlation structures were tested using likelihood ratio tests $\alpha = 0.05$ (Garber and Maguire 2003). All generalized nonlinear models were fit using the nlme3 library (Pinheiro and Bates 2000) in S-PLUS 7.0 (Insightful Corporation, Seattle, Washington).

Table 4. Parameter estimates and asymptotic SEs for Equation 2 for managed red alder.

Parameter	Estimated value	SE
a_1	0.8995	0.0281
a_2	1.0205	0.0107
a_3	0.2631	0.0050
a_4	−18.8990	1.7281
a_5	4.2549	0.0902
a_6	0.6221	0.0311

Table 5. Estimated average bias and SEE of dib by disc position (height) for Equation 2 for the modeling data.

Disc position	n	Bias (in)	Bias (%)	SEE (in.)	SEE (%)
0.6 ft	234	0.058	0.756	0.575	7.465
1.6 ft	234	0.023	0.321	0.331	4.714
2.6 ft	234	0.011	0.171	0.232	3.468
4.5 ft	234	0.018	0.279	0.067	1.055
H10%	234	−0.012	−0.204	0.251	4.270
H20%	234	−0.041	−0.759	0.267	4.910
H30%	234	0.001	0.023	0.289	5.766
H40%	234	0.016	0.351	0.341	7.634
H50%	234	0.002	0.049	0.338	8.866
H60%	234	0.038	1.195	0.335	10.663
H70%	234	0.009	0.403	0.314	13.429
H80%	234	−0.015	−0.976	0.243	16.046
H90%	234	−0.012	−1.587	0.156	21.322
H95%	234	0.012	3.014	0.122	31.829
All	3,276	0.008	0.178	0.296	6.850

Evaluation of Equation 2 consisted of an adjusted coefficient of determination (Kvålseth 1985) and the assessment of the estimated average bias and standard error of the estimate (SEE) in predicting dib among relative heights and in predicting volume among tree diameter classes using both the modeling and the evaluation data sets. Comparisons of dib and volume prediction also were made among several existing red alder taper (Curtis et al. 1968, Kozak 1988, 1997) and volume equations (Browne 1962, Snell and Little 1983) using the evaluation data set. Average bias (B) and SEE were calculated as follows:

$$B = \frac{1}{n} \sum_{i=1}^{n} (Y_i - \hat{Y}_i)$$

$$\mathrm{SEE} = \sqrt{\frac{\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2}{n - k}} \qquad (3)$$

where Y_i = observed dib or volume, $\hat{Y}_i$ = predicted dib or volume, n = the number of observations, and k = the number of model parameter estimates. Observed tree volumes were determined by applying Smalian's formula to the observed dib measures for each tree and predicted volumes were determined by analytically (Curtis et al. 1968) or numerically (Kozak 1988, 1997, Equation 2) integrating the taper model from stump (0.5 ft) to tree tip.

Results

Equation 2 fit well, with a root mean square error of 0.21 in. and an adjusted coefficient of determination of nearly 0.99. Parameter estimates for Equation 2 are presented in Table 4. Overall average dib bias for Equation 2 was 0.008 in. (Table 5). Along the stem, average biases generally were positive, indicating a slight underestimate of observed dib on average. The largest underestimate occurred at 0.6 ft; however, the average percent bias at that location was well within the range of the other average percent biases along the stem. With the exception of the 0.6-ft observations, the SEEs were all well under 0.5 in. Estimated average bias in stem volume across all tree diameter classes for Equation 2 was very small, nearly 0.1 ft^3 (0.9%). There was a slight overestimate of volume in the smallest trees and an increasing underestimate at progressively larger trees. However, the estimated average percent bias was fairly constant among tree dbh classes (Table 6).

Table 6. Estimated average bias and SEE of total volume by diameter class for Equation 2 for the modeling data.

Diameter class (in.)	*n*	Bias (ft^3)	Bias (%)	SEE (ft^3)	SEE (%)
3.0–5.0	34	−0.011	−1.588	0.166	8.242
5.0–7.0	106	0.102	1.639	0.343	7.761
7.0–9.0	68	0.092	0.598	0.484	6.223
9.0–11.0	26	0.237	1.601	0.996	7.971
All	234	0.095	0.863	0.452	7.132

The comparisons among the existing taper and volume equations and Equation 2 on the evaluation data revealed some interesting patterns. The two existing variable exponent models had the largest average biases at the lower stem positions (Table 7). The extremely large average bias produced by Kozak's 1994 equation (Kozak 1997) was caused by large overestimates of groundline dib measurements. However, at higher stem positions these equations did very well. Equation 2 and Curtis et al. (1968) did well at lower stem positions but average biases increased at midstem positions and were higher for the Curtis et al. (1968) equation. Average volume biases generally were small for all the equations except the large overestimates in volume and the high SEEs for Snell and Little's (1983) equation (Table 8). All models overestimated volume for the 5.0- to 10-in. dbh class. In contrast, Kozak's two variable exponent equations resulted in underestimates in the trees under 5.0 in.

Table 7. Estimated average bias and SEE of dib by relative height class for Equation 2, and several published taper equations for red alder for the plantation evaluation data.

Relative height class	*n*	Bias (in.)	Bias (%)	SEE (in.)	SEE (%)
		Equation 2			
0.0–0.2	96	0.071	1.186	0.521	8.757
0.2–0.4	52	−0.248	−5.770	0.441	10.284
0.4–0.6	43	−0.412	−13.011	0.694	21.915
0.6–0.8	38	−0.308	−14.851	0.577	27.849
0.8–1.0	28	−0.297	−37.286	0.434	54.400
All	257	−0.171	−4.251	0.513	12.778
		Curtis et al. 1968			
0.0–0.2	96	−0.070	−1.182	0.556	9.346
0.2–0.4	52	−0.320	−7.462	0.471	10.974
0.4–0.6	43	−0.618	−19.526	0.890	28.092
0.6–0.8	38	−0.642	−30.966	0.925	44.627
0.8–1.0	28	−0.674	−84.498	0.866	108.573
All	257	−0.363	−9.042	0.666	16.584
		Kozak 1988			
0.0–0.2	96	0.368	6.184	0.724	12.169
0.2–0.4	52	−0.154	−3.595	0.363	8.457
0.4–0.6	43	−0.485	−15.317	0.779	24.583
0.6–0.8	38	−0.331	−15.979	0.693	33.406
0.8–1.0	28	−0.132	−16.536	0.407	50.974
All	257	−0.038	−0.955	0.604	15.048
		Kozak 1994[a]			
0.0–0.2	96	−4.460	−74.980	16.106	270.756
0.2–0.4	52	0.222	5.173	0.369	8.603
0.4–0.6	43	−0.151	−4.781	0.507	16.001
0.6–0.8	38	−0.297	−14.317	0.550	26.545
0.8–1.0	28	−0.275	−34.421	0.462	57.964
All	257	−1.720	−42.864	9.509	236.936

[a] From Kozak 1997.

Table 8. Estimated average bias and SEE of total volume by diameter class for Equation 2 and Kozak (1988) for the plantation evaluation data.

Diameter class (in.)	*n*	Bias (ft^3)	Bias (%)	SEE (ft^3)	SEE (%)
		Equation 2			
0.0–5.0	14	−0.014	−1.118	0.180	13.083
5.0–10.0	26	−0.102	−2.280	0.385	8.672
All	40	−0.071	−2.113	0.307	9.142
		Browne 1962[a]			
0.0–5.0	14	0.021	1.620	0.159	11.193
5.0–10.0	26	−0.258	−5.776	0.470	10.442
All	40	−0.159	−4.702	0.381	11.119
		Curtis et al. 1968			
0.0–5.0	14	−0.053	−3.881	0.212	15.341
5.0–10.0	26	−0.290	−6.525	0.516	11.563
All	40	−0.208	−6.145	0.406	12.078
		Snell and Little 1983[a]			
0.0–5.0	14	−1.769	−125.028	1.946	137.354
5.0–10.0	26	−2.041	−45.416	2.624	58.419
All	40	−1.946	−56.973	2.355	68.941
		Kozak 1988			
0.0–5.0	14	0.117	8.340	0.279	20.068
5.0–10.0	26	−0.046	−1.002	0.420	9.465
All	40	0.011	0.338	0.339	10.010
		Kozak 1994[b]			
0.0–5.0	14	0.300	21.567	0.632	45.650
5.0–10.0	26	−0.194	−4.407	1.360	30.589
All	40	−0.021	−0.680	1.021	30.219

[a] Total stem volume equations from groundline to tree tip

[b] From Kozak 1997.

Discussion

The equation and the associated merchantable volume table (Table 9) presented here are the only known volume estimation tools developed using red alder plantation data. The model predicted dib and volume well with a relatively high degree of precision for both the modeling data set and the limited plantation data available to the authors. Testing the equation on larger trees would have been desirable but larger plantation-grown tree data do not exist.

The performance of existing variable exponent equations developed in natural red alder stands on this limited plantation data was surprising. These equations were generally superior to the other equations in predicting upper stem diameters and volume. Kozak's (1988) equation predicted dib and volume accurately and precisely. The Kozak 1994 equation (from Kozak 1997) did well in predicting upper stem diameters. However, it produced large overpredictions at lower stem positions because of a large flare in the lowest 10% of the stems in trees with large dbh/ht ratios. This translated into higher average biases and lower precision in volume predictions. Likewise, this was the case with many of the preliminary models evaluated by the authors using the modeling data that necessitated the alternative parameterization of *C*. Management activities, such as initial planting density, pruning, and thinning, may exacerbate this problem in these plantations. Management has a direct effect on stand conditions that have been shown to affect stem shape of many tree species (Larson 1963). Managed red alder stands typically have trees with greater dbh/ht ratios (caused mainly by increases in dbh

Table 9. Merchantable stem volume in cubic feet (5-in. dob bark top, 6-in. stump, minimum 10-ft log) based on total tree height and dbh.

DBH (in)	Total tree height (ft)												
	25	30	35	40	45	50	55	60	65	70	75	80	85
6	1.43	**1.58**	**1.74**	**1.89**	**2.04**	**2.19**	**2.34**	**2.49**	**2.63**	2.78	2.92	3.06	3.20
7	2.35	2.70	**3.05**	**3.41**	**3.78**	**4.14**	**4.51**	**4.88**	**5.25**	**5.63**	6.00	6.37	6.75
8	–	3.80	4.34	4.89	**5.46**	**6.02**	**6.60**	**7.18**	**7.76**	**8.35**	8.94	9.53	10.12
9	–	–	5.68	6.43	**7.19**	**7.97**	**8.75**	**9.54**	**10.34**	**11.14**	11.95	12.76	13.58
10	–	–	–	8.06	9.03	**10.02**	**11.02**	**12.03**	**13.05**	**14.08**	15.12	16.16	17.22
11	–	–	–	–	10.98	12.20	13.43	**14.67**	**15.93**	17.20	18.48	19.77	21.07
12	–	–	–	–	–	14.51	15.99	17.48	18.99	20.51	22.05	23.60	25.16
13	–	–	–	–	–	16.97	18.70	20.46	22.23	24.02	25.83	27.66	29.50

Sample distribution is indicated by bold typeface.

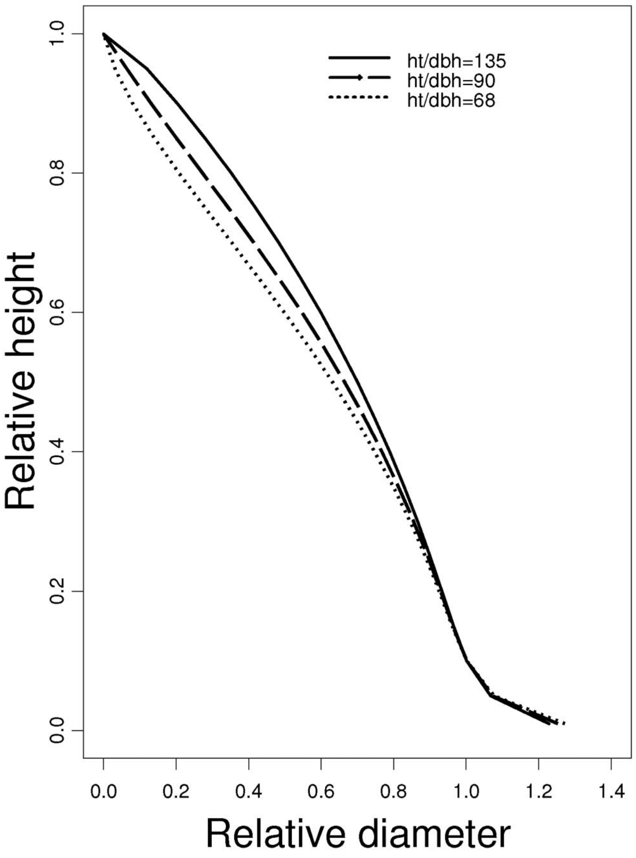

Figure 1. Stem profiles for three trees with ht/dbh of 135, 90, and 68 (45 ft in height with a dbh of 4, 6, and 8 in., respectively).

growth) than generally found in unmanaged stands because of, primarily, density management (Hibbs et al. 1989, Puettmann 1994, DeBell and Harrington 2002).

Equation 2 also suggests a large change in stem shape with ht/dbh (Figure 1). The relative stem profiles of three trees of equal height (45 ft) but of different diameters (4-, 6-, and 8-in. dbh) indicate that trees with a low ht/dbh are more parabolic than trees with a high ht/dbh. This change in stem shape results in an increase in merchantable height and greater log recovery for trees with a low ht/dbh than would be predicted by the existing red alder volume equations. The model presented here (and associated volume estimates) account for these stem form changes. This model therefore, is sensitive to stand conditions and/or management activities that affect stem form and provides better predictions than existing models for the plantation-grown red alder. Thus, it can be used as a general replacement in stand volume inventories and could potentially be incorporated into future red alder growth and yield models.

Endnote

[1] The value 1.364409 was the product of unit transformation as the data were fit in metric units and converted to imperial units.

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