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Height models for Red Alder (*Alnus rubra* Bong.) in British Columbia

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Application. The height-bha and years to breast height models will provide growth and yield information that is useful for developing silviculture prescriptions and for forest management planning. As red alder is fast growing, this information is particularly critical for evaluating early stand performance. The models will also play a role in updating inventories.

Abstract. Interest in managing red alder (*Alnus rubra* Bong.) for pulp, lumber and improving site quality is increasing. Proper management of this species requires growth and yield information. Height-age and years to breast height models developed for western Washington and northwestern Oregon are being used in British Columbia. It is unknown whether these models are suitable for British Columbia's conditions. Therefore, new models were developed from thirty stem analysis plots established in red alder stands in coastal areas of British Columbia. The sample trees in the plots were intensively sampled by taking sections out of the tree stems and counting the annual growth rings on each section. This led to height-age data for each tree, which was then averaged by age to get plot height-age data. Years to breast height and height-age models were developed from these data and are summarized in this paper. The height-age model was anamorphic, which contrasts with the model previously used in British Columbia. The new years to breast height model provides more resolution than the previous model.

Key words: breast height age, dominant height, height-age, site index, years to breast height

Introduction

Red alder (*Alnus rubra* Bong.) is a fast-growing deciduous species that occurs in the Pacific Northwest region of North America. It ranges from southern California to southeastern Alaska, mainly within 200 km of the Pacific Ocean and at lower elevations, particularly in the northern part of its range (Harrington et al. 1994). It is found in pure and mixed stands, often with Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco var. *menziesii*), western hemlock (*Tsuga heterophylla* (Raf.) Sarg.), western redcedar (*Thuja plicata* Donn.), and Sitka spruce (*Picea sitchensis* (Bong.) Carr.) (Peterson et al. 1996). In mixed stands, alder is usually accompanied by shade-tolerant conifers because shade-intolerant species cannot survive under the fast growing alder. However, alder can be found with intolerant conifers if the conifer is established first or is in gaps in the alder stand. Red alder's life span is short, usually less than 100 years, and hence it is often replaced by conifers (Harrington et al. 1994).

In British Columbia, red alder is becoming increasingly valuable for lumber and for other values such as biodiversity and long-term site productivity (Peterson et al. 1996). There is also interest in using alder for pulp (Hrutfiord 1978) and particleboard (Maloney 1978). Alder improves long-term productivity by fixing atmospheric nitrogen (Binkley et al. 1994). This property can be utilized by managing red alder with other species on nitrogen-deficient sites, thereby increasing the productivity of the site for the companion species (Hibbs and DeBell 1994). One of the major factors to be considered in managing red alder in mixed stands is the growth rate of each species in the stand (Peterson et al. 1996).

Accurate height-breast height age (bha) models are required for predicting the growth and yield of trees and stands. As alder is much faster growing in youth and shorter-lived than coniferous species commonly found in coastal British Columbia, it is important to have good height-bha models for alder in order to capture these growth differences. Height-age models for red alder developed by Harrington and Curtis (1986) from data collected in Oregon and Washington have been used in British Columbia up to now. However, there may be some differences in height growth patterns between British Columbia and Oregon/Washington because of differences in sites. Also, the data from the Oregon/Washington study were collected using standards and procedures that may not be compatible with the standards used in British Columbia. The Harrington-Curtis models were based on total age and the site index had a base age of 20 years total age, whereas breast height age models with a base age of 50 years are the norm for British Columbia. Therefore, new height-bha models that are applicable in British Columbia were developed. A years to breast height model was also developed so that total age could be converted into bha and vice versa. Previously, it was assumed that it took two years to achieve breast height from germination (Bishop et al. 1958). The years to breast height function is important because, among other reasons, it is used to estimate the time to green-up, which is an important parameter in timber supply analyses. The development of these models is reported here.

Table 1. Summary information about the 30 stem analysis plots

Variable	Mean	Minimum	Maximum
Breast height age (yrs)	36	25	54
Height (m)	22.67	14.77	27.84
Site index (m @ bha 25)	20.40	12.79	25.35
Years to breast height (yrs)	1.8	0.5	3.8

Data

The data for this project consists of 30 stem analysis plots located in pure red alder stands in southern coastal British Columbia. The plots were located in the Coastal Western Hemlock (CWH) dry maritime (dm), very dry maritime (xm), and submontane very wet maritime (vm1) biogeoclimatic units (Green and Klinka 1994). These units likely provide the best climatic conditions for red alder in British Columbia and hence were targeted for sampling. The nutrient regimes of the plots were either rich or very rich and their moisture regimes were slightly dry, fresh, or moist. Plot density ranged from 350 stems per ha to 1925 stems per ha. The data are described in more detail in Courtin (1992). Seven more plots were stem analyzed but they were below the index age of 25 years bha so they were not included in this analysis. Three dominant undamaged site trees were selected for stem analysis from each 0.04 ha plot. The sample trees were sectioned at ground level, 0.7 m, 1.3 m, and at 1 m intervals up to 15 m, and then at 2 m intervals above 15 m. The rings at each section were counted, resulting in a set of height-ring count data for each sample tree. Table 1 presents summary information about the stem analysis plots. These data cover approximately the same range of sites and ages as does the Harrington and Curtis (1986) data, although their dataset contained eight more plots than ours.

Methods

Height-bha data were generated from the height-ring count data by first removing the stem analysis bias using the technique described by Newberry (1991). This bias arises because the sectioning points do not occur at the end of a year’s growth. The heights of the three trees in each plot were averaged by bha up to the age of the youngest tree in the plot. This resulted in a set of dominant height-bha data for each plot. Site index (SI_{25}) is defined as dominant height of the plot at bha 25. Therefore, site index for red alder has a base age of 25 years bha. The height-bha trajectories were graphed to detect suppression and/or damage. No suppression or damage was found, but it was not expected to be a problem since alder is so fast growing that it is not often overtopped and because its survival rate is low even under partial shade (Peterson et al. 1996).

The years to breast height data were generated by subtracting each tree's ring count at breast height from the ring count at ground level and then adding 0.5 to remove the stem analysis bias. The years to breast height for each tree in a sample plot were averaged to give a years to breast height for the plot.

Two models were developed from these data: a years to breast height model and a height-bha model. The years to breast height model predicts the number of years it takes red alder to reach breast height from seed as a function of site index. The height-bha model predicts the dominant height trajectory for a given site index. The functional form of the years to breast height and height-bha models are given in equations (1) and (2), respectively. The term beginning with coefficient b_2 in equation (2) was included as a test for polymorphic height growth.

$$\mathrm{YTBH} = \mathrm{b}_0 + \mathrm{b}_1 \times \mathrm{SI}_{25} + \varepsilon \tag{1}$$

$$\mathrm{H} = 1.3 + (\mathrm{SI}_{25} - 1.3) \times \frac{1 + e^{\mathrm{b}_0 + \mathrm{b}_1 \times \ln(24.5) + \mathrm{b}_2 \times \ln(\mathrm{SI}_{25} - 1.3)}}{1 + e^{\mathrm{b}_0 + \mathrm{b}_1 \times (\mathrm{BHA} - 0.5) + \mathrm{b}_2 \times \ln(\mathrm{SI}_{25} - 1.3)}} + \varepsilon \tag{2}$$

where: YTBH = number of years it takes to reach breast height from germination,
SI_{25} = site index (m at bha 25),
H = dominant height (m),
BHA = breast height age (years),
b_0, b_1, b_2 = model parameters (not necessarily the same value for both models), and
ε = random error term.

Model (1) was fit to the data using linear least squares regression and model (2) was fit with nonlinear least-squares regression. In the analysis of model (2), the observation corresponding to bha 25 was deleted from the data because the conditioning of the model at bha 25 makes these data points redundant.

The following tests were done to evaluate how well the standard regression assumptions were met:

1. H_0: residuals have a mean of zero – t-test
2. H_0: residuals are normally distributed – Shapiro and Wilk's (1965) W test
3. H_0: residuals are homoscedastic – residual plots.

Table 2. Results of the model analyses

	Parameter estimates		Mean squared		
Model	b_0[a]	b_1[a]	error	R squared	W[b]
1	5.494 (1.001)	–0.1789 (0.04869)	0.4578	0.3252	0.9578 (0.3117)
3	3.600 (0.06333)	–1.240 (0.04137)	0.9668	0.9771	0.9931 (0.9902)

[a] standard errors of the estimated parameters are shown below the parameter estimates.
[b] p-values of the statistics are shown below the statistics.

In addition to the above tests, model (2) was evaluated for nonlinear behaviour by testing the intrinsic and parameter-effects nonlinearity (Bates and Watts 1980, Ratkowsky 1983) and parameter bias (Box 1971). A model that behaves close to linearly is important for many reasons (Ratkowsky 1983). Linear behaviour is particularly important in this analysis because the parameters were tested for significance with a t-test. This test becomes more valid as the parameter-effects nonlinearity is reduced (Ratkowsky 1983). The guidelines given by Ratkowsky (1983) were used to test whether the intrinsic and parameter-effects nonlinearity and parameter bias were significant.

Results

Parameter b_2 in the height-bha model (model (2)) was not significantly different from zero. Therefore, the term associated with parameter b_2 was deleted from the model, resulting in model (3), which was then analyzed.

$$H = 1.3 + (SI_{25} - 1.3) \times \frac{1 + e^{b_0 + b_1 \times \ln(24.5)}}{1 + e^{b_0 + b_1 \times \ln(BHA - 0.5)}} + \varepsilon \qquad (3)$$

Results of the analysis of models (1) and (3) are summarized in Table 2. This table presents the estimated model parameters and their standard errors, the mean squared error, R^2 statistic, and the W test for normality and its significance level. Figures 1 and 2 are graphs of the residuals of models (1) and (3), respectively, plotted against selected variables. The plots of the residuals for model (1) did not show evidence of heteroscedasticity. The plots of the residuals for model (2) indicated that the variance may vary with bha; however, it was not deemed to be serious. This pattern in the variance is typical

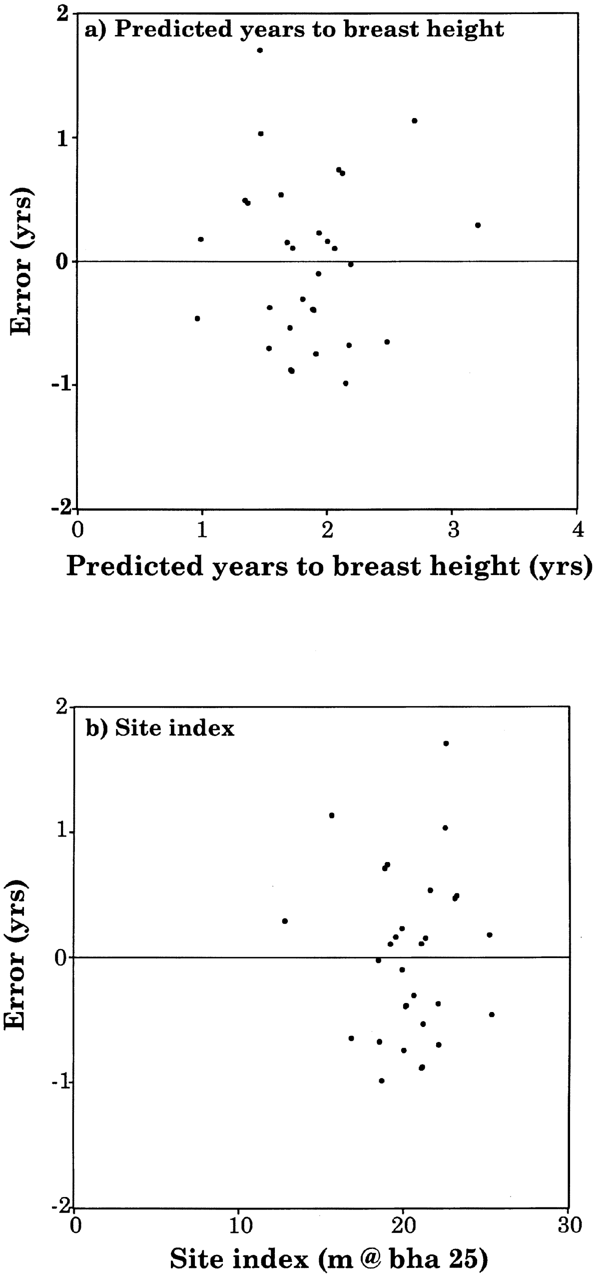

Figure 1. Residuals from model (1) plotted against: (a) predicted years to breast height, and (b) SI_{25}.

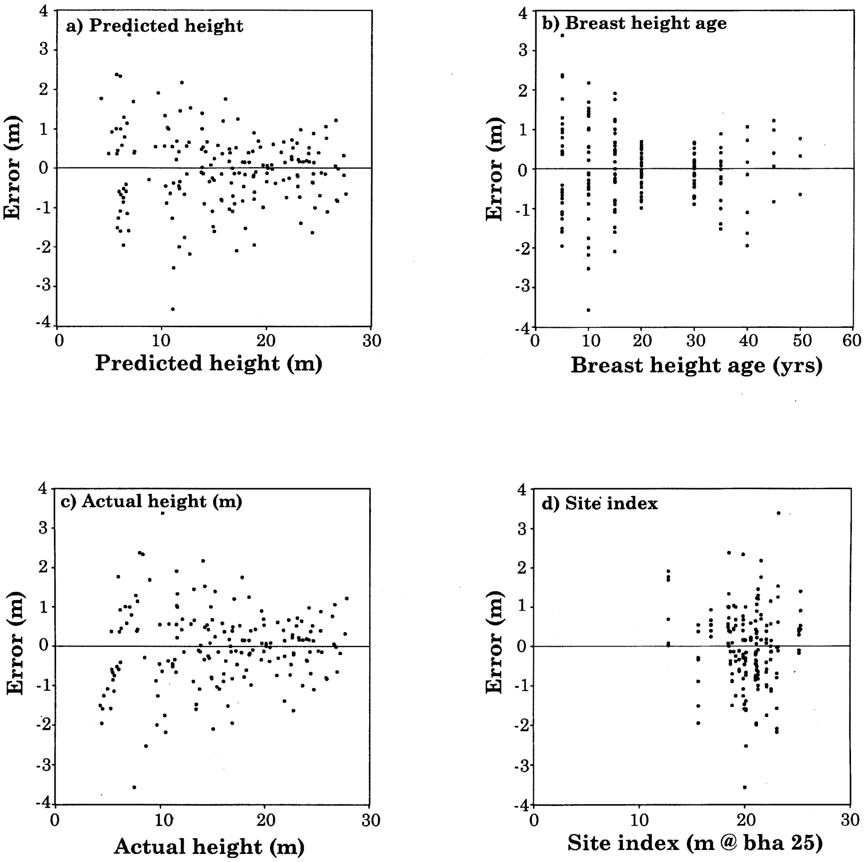

Figure 2. Residuals from model (3) plotted against: (a) predicted heights, (b) bha, (c) actual height, and (d) SI_{25}.

of height-age models, particularly those that are constrained to pass through the site index (Nigh and Sit 1996).

The biases in parameters b_0 and b_1 of model (3) were less than 1% and the intrinsic and parameter-effects nonlinearity were small. Therefore, based on the criteria given by Ratkowsky (1983), we concluded that the model behaved close to linearly.

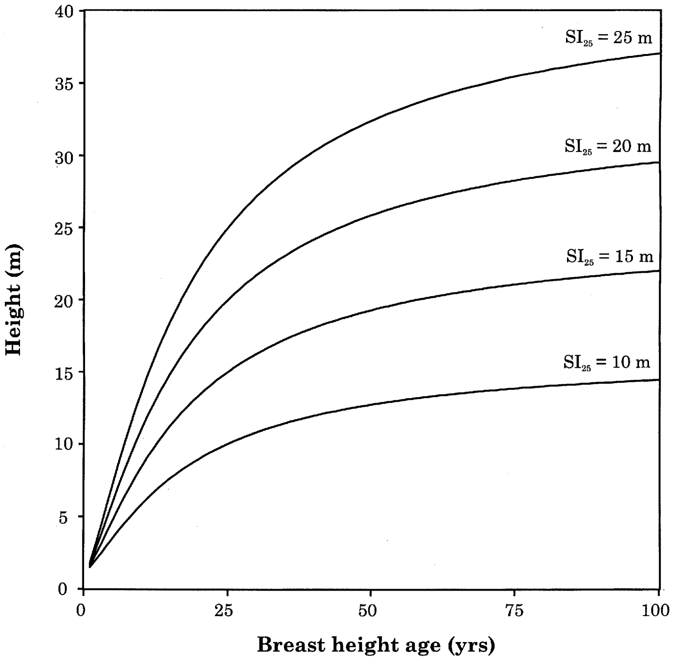

Figure 3. Model (3) plotted for SI_{25}'s of 10, 15, 20, and 25.

Discussion

The new years to breast height and height-bha models are given in equations (4) and (5), respectively. The years to breast height should be set to 1.0 when SI_{25} is greater than 25 to avoid unrealistic estimates.

$$\begin{aligned} \text{YTBH} &= 5.494 - 0.1789 \times SI_{25} && \text{if } SI_{25} \leq 25, \\ &= 1.0 && \text{otherwise.} \end{aligned} \tag{4}$$

Model (5) is an algebraically simplified form of (3). Figure 3 shows the corresponding height-bha curves.

$$H = 1.3 + \frac{1.693 \times (SI_{25} - 1.3)}{1 + e^{3.600 - 1.240 \times \ln(\text{BHA} - 0.5)}} \tag{5}$$

Model (5) can be rearranged to allow SI_{25} to be explicitly estimated from dominant height and bha (equation (6)).

$$SI_{25} = 1.3 + (H - 1.3) \times (0.5906 + 21.61 \times e^{-1.240 \times \ln(\text{BHA} - 0.5)}) \tag{6}$$

A conversion from SI_{25} to a site index with an index age of 50 years (SI_{50}) can also be derived (equation (7)).

$$SI_{50} = -0.4063 + 1.313 \times SI_{25} \tag{7}$$

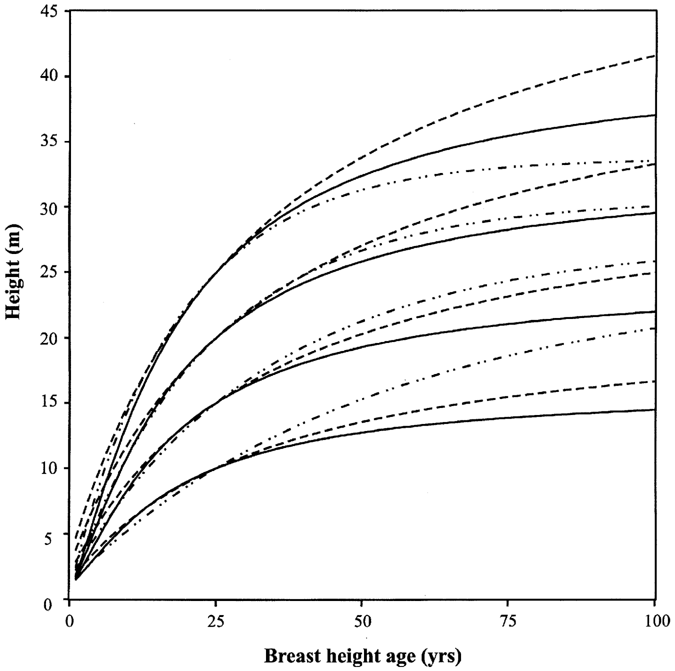

Figure 4. Comparison of model (3) (——————), the Harrington and Curtis (1986) model (—— —— ——), and the Bishop et al. (1958) (—— ·· —— ·· ——) model.

The new height-bha models for red alder are anamorphic, that is, the estimated heights for any two site indices are proportionally the same at all ages. This is contrary to the height growth patterns exhibited by many species, which show sigmoidal height growth on better sites and nearly linear growth on lower sites (Avery and Burkhart 1994). Bishop et al. (1958) also found that the height growth of red alder was anamorphic. This was before polymorphic growth became widely modelled so the anamorphism may be due to the method of analysis rather than observed growth. Since model (2) allows for both polymorphic and anamorphic height growth, the data rather than the method of analysis determined the anamorphic growth pattern. The height-age models developed by Harrington and Curtis (1986) are polymorphic. These three models are graphed up to bha 100 in Figure 4. Note that the range of the data does not extend much past bha 50, so differences past this age may be attributable to the functional form of the model rather than height growth pattern. Nevertheless, the behaviour of the models in extrapolated age ranges is important and hence are shown (Nigh 1997). The models were conditioned to go through heights 10, 15, 20, and 25 m at bha 25 to determine where the differences lay. In general, the new model is closer to the Bishop et al. model

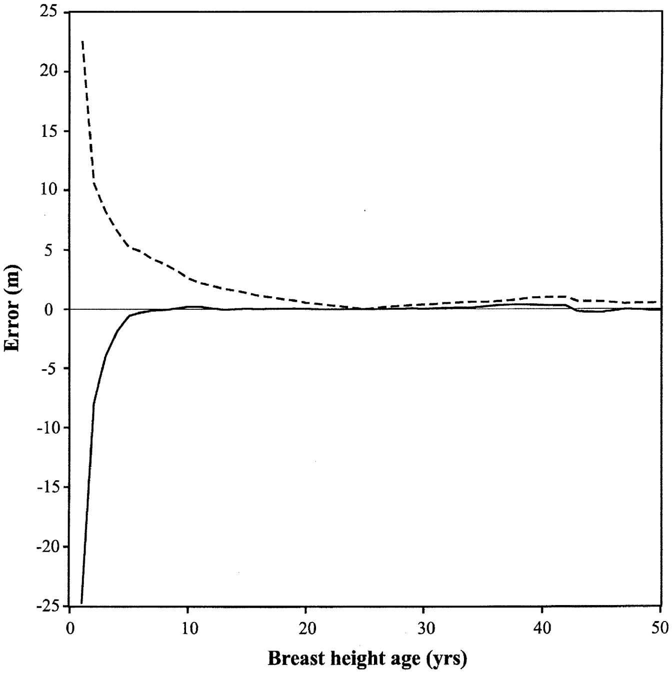

Figure 5. Mean error (——————) and the standard deviation of the errors (—— —— ——) in site index estimates from models (5)/(6).

below the index age but closer to the Harrington/Curtis model near bha 50. The new model shows slower growth than the other two models in the extrapolation range, i.e., above bha 50, except on better sites. This is a desirable trait since red alder is short-lived and its height growth is mostly finished by age 50 (Peterson et al. 1996). The differences in the height growth patterns, particularly those at young ages, indicate that our model should be better suited than the other two models for red alder stands in British Columbia.

Model (6) was obtained by inverting model (5). This results in the site index estimates from the two models being identical. These site index estimates are, however, not optimal and grossly inaccurate estimates of site index can be obtained from young stands. Figure 5 shows the mean error and the standard deviation of the error in the site index estimates from models (5) and (6). These models should not be used to estimate site index for bhas less than about 5 if the mean error is a concern, or less than around 10 years bha if the variability in the site index estimates is a concern.

Implementing the new height-age and years to breast height models will have some growth and yield implications. Estimated years to breast height will be longer for poorer sites, but shorter for better quality sites. Estimates of stand volume will not be impacted substantially below breast height age 40 or 50 because the Harrington/Curtis and the new height-age models give similar height predictions at these ages. At older ages, however, estimated volumes will be lower with the new height-age model because it gives shorter height estimates.

Red alder is noted for being exceptionally fast-growing but, compared to its cohorts and especially with its conifer cohorts, it is short-lived. Its height growth is rapid during the juvenile stage but slows after this stage. This growth pattern is evident in the new height-bha model as shown in Figure 3. Height growth is rapid up to bha 25 to 30, then slows markedly, particularly on poorer sites.

Conclusion

Red alder's commercial and biological importance is increasing. Good growth and yield information is required for the proper management of this species. The height-bha and years to breast height models reported here are better suited to conditions in British Columbia because the data were collected from local sites and to local standards. In addition, the years to breast height function has more resolution than previous functions. These new models should be used for estimating red alder height and site index in British Columbia.

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