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Volume and Taper Tables For Red Alder

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INTRODUCTION

Red alder is a species of increasing importance in the Pacific Northwest. Interest in improved volume tables for the species resulted in publication of the 1949 volume tables (Johnson et al, 1949; Skinner 1959). More recently, Browne (1962) published cubic-foot-volume tables for red alder in British Columbia, and Hoyer (1966) presented tarif access tables, based on the 1949 tables, for use with the general tarif tables of Turnbull et al. (1963).

This paper presents new volume and taper tables for red alder. These form a unified system, permitting computation of cubic- and board-foot volumes for alternative assumed scaling practices and limits of merchantability.

DATA

Basic data used in construction of these tables consisted of (1) tree measurement data from Oregon, Washington, and British Columbia used in construction of the 1949 tables; (2) additional tree measurement data from Washington and Oregon contributed by Weyerhaeuser Co.; and (3) tree measurement data originally used for the volume tables published by Johnson et al. in 1926. After elimination of trees lacking measurements of upper stem diameters to less than 10 inches, having low forks or other gross abnormalities or apparent errors in recording of data, a total of 473 trees were available for the analysis (table 1).

METHODS

A report on the method of data analysis is planned for separate publication. Therefore, only an outline of the approach used is given in this paper.

Basically, the method consisted in deriving from the pooled tree measurement data an equation expressing squared diameter inside bark — $(d.i.b.)^2$ — as a function of the relative height at point of measurement, diameter outside bark at breast height (d.b.h.) and total height (H) of the tree.[1]

Cubic volumes (CV) were then estimated by integration of the resulting equation. Estimates of upper stem diameter were obtained as the square root of the estimated $(d.i.b.)^2$. Board-foot volumes were estimated by applying assumed merchantability and scaling standards to the estimates of upper stem diameters. Equations were then developed for conversion of total cubic volume to merchantable cubic- and board-foot volumes.

[1] Since the principal intended use of the equation was in estimation of volume, using the relationship cubic volume $= k \int (d.i.b.)^2 dH$,
$(d.i.b.)^2$ rather than d.i.b. was considered the variable of primary interest.

RESULTS

CUBIC-VOLUME TABLE

Table 2 gives estimates of average cubic volume of red alder trees of specified d.b.h. and total height. Volumes are inside bark, from a stump of height in feet equal to [½ + (½)(d.b.h./12)],[2] to the indicated inside bark (i.b.) top diameter limits of 12.0, 10.0, 8.0, 6.0, 4.0, and 0.0 inches. No trim allowance is made.

These estimates were obtained by integration of the estimating equation for $(d.i.b)^2$ from assumed stump height to heights corresponding to the indicated diameter limits.

TAPER TABLE

Table 3 gives estimated diameters inside bark of red alder trees of specified d.b.h. and total height, at intervals of 4.125 feet above stump height. Values given are square roots of the values of $(d.i.b.)^2$ predicted by the estimating equation. The height interval used provides trim allowance of 0.50 foot per 16 feet of bole.

VOLUME TABLES IN BOARD FEET

Tables 4, 5, and 6 give estimated average tree volume in board feet Scribner (SV) for stated assumed scaling lengths (8 and 16 feet), top diameter limits (12.0, 10.0, 8.0, and 6.0 inches i.b.), and diameter measurement practice ("nearest inch" versus rounding of fractional diameters to next lower exact inch). Volumes were obtained by applying the formula Scribner log rule (Bruce 1925) to estimated upper stem diameters (table 3), with trim allowance of 0.50 foot per 16 feet of merchantable stem and using, for top logs, estimated length (unrounded) to the stated top d.i.b. limit.

Use of formula rule rather than scale stick values eliminates the arbitrary and illogical jumps which result from rounding and the diagraming process. Volumes for measurement to "nearest inch" were obtained by using estimated top d.i.b. in the formula; volumes for measurement "rounding down" were obtained by using (d.i.b.—½) as the diameter in the formula. Since volume tables represent the means of many trees, smoothed values are appropriate.

Table 7, which gives estimated Scribner volumes by d.b.h. class and number of 16-foot logs, was prepared by graphical interpolation from values shown in tables 3 and 5.

Table 8 gives estimated tree volumes in board feet by International ¼-inch rule, with the same assumed measurement practices used in table 4.

CHOICE OF TOP DIAMETER LIMIT

In application of either board-foot or merchantable cubic-volume tables, the choice of minimum top d.i.b. limit should be based on the user's estimate of the top d.i.b. to which average trees of the size under consideration can actually be utilized. This limit will vary with tree size, because of excessive branching in the tops of large trees. Thus, board-foot volume to a 6-inch top in a tree 24 inches d.b.h. is not a realistic estimate of the volume of material actually utilizable as sawtimber, even though smaller trees may be utilized to this limit.

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[2] An arbitrary rule, which approximates average height of stumps as cut.

COMPARISON OF "ACTUAL" AND ESTIMATED VOLUMES

"Actual" cubic volume of sample trees was calculated by Smalian's formula (except with butt section treated as a neiloid) and the differences $[CV_{estimate} - CV_{actual}]$, where estimated volume was CVT as shown in table 2, were expressed as percentages of estimated volume. The mean percentage difference, excluding 131 trees for which stump diameter had not been recorded, was –1 percent with standard deviation of 9 percent.

For a portion (139) of the sample trees, differences between estimated Scribner volume (SV8 in table 6) and "actual" Scribner volume were calculated. These, when expressed as percentages of estimated volume, had a mean of –0.8 percent and standard deviation of 15 percent.

TARIF ACCESS CURVES

Those wishing to use the tarif system for volume computation (Turnbull et al. 1963) can readily use table 2 as the basis for a tarif access table by means of the relationship

$$\text{tarif number} = \frac{0.913(\text{CV4})}{(\text{basal area} - 0.087)}$$

given by Turnbull and Hoyer (1965).

Figure 1 is such an access table in graphical form.

COMPUTING EQUATIONS

The equations given below may be used for calculations on automatic data processing equipment.

1. Diameters inside bark may be estimated by the following equation, which is the basis of table 3.

$$\text{d.i.b.}_{est} = (\text{d.b.h.})\sqrt{\left[\frac{(\text{d.i.b.})^2}{(\text{d.b.h.})^2}\right]_{est}}$$

where

$$\left[\frac{(\text{d.i.b.})^2}{(\text{d.b.h.})^2}\right]_{est} = 0.91274(x)^{1.5} - 1.9758(x^{1.5} - x^3)(\text{d.b.h.})(10^{-2})$$
$$+ 8.2375(x^{1.5} - x^3)(H)(10^{-3}) - 4.964(x^{1.5} - x^{32})(H)(\text{d.b.h.})(10^{-5})$$
$$+ 3.773(x^{1.5} - x^{32})(H)^{0.5}(10^{-3}) - 7.417(x^{1.5} - x^{40})(H)^2(10^{-6})$$

in which

H = total height of tree,

d.b.h. = diameter outside bark at breast height,

and

$$x = \frac{(\text{distance from tip to estimated d.i.b.})}{(H - 4.5)}$$

The ratio $(\text{d.i.b.})^2/(\text{d.b.h.})^2$ has a standard error of estimate of approximately 0.07.

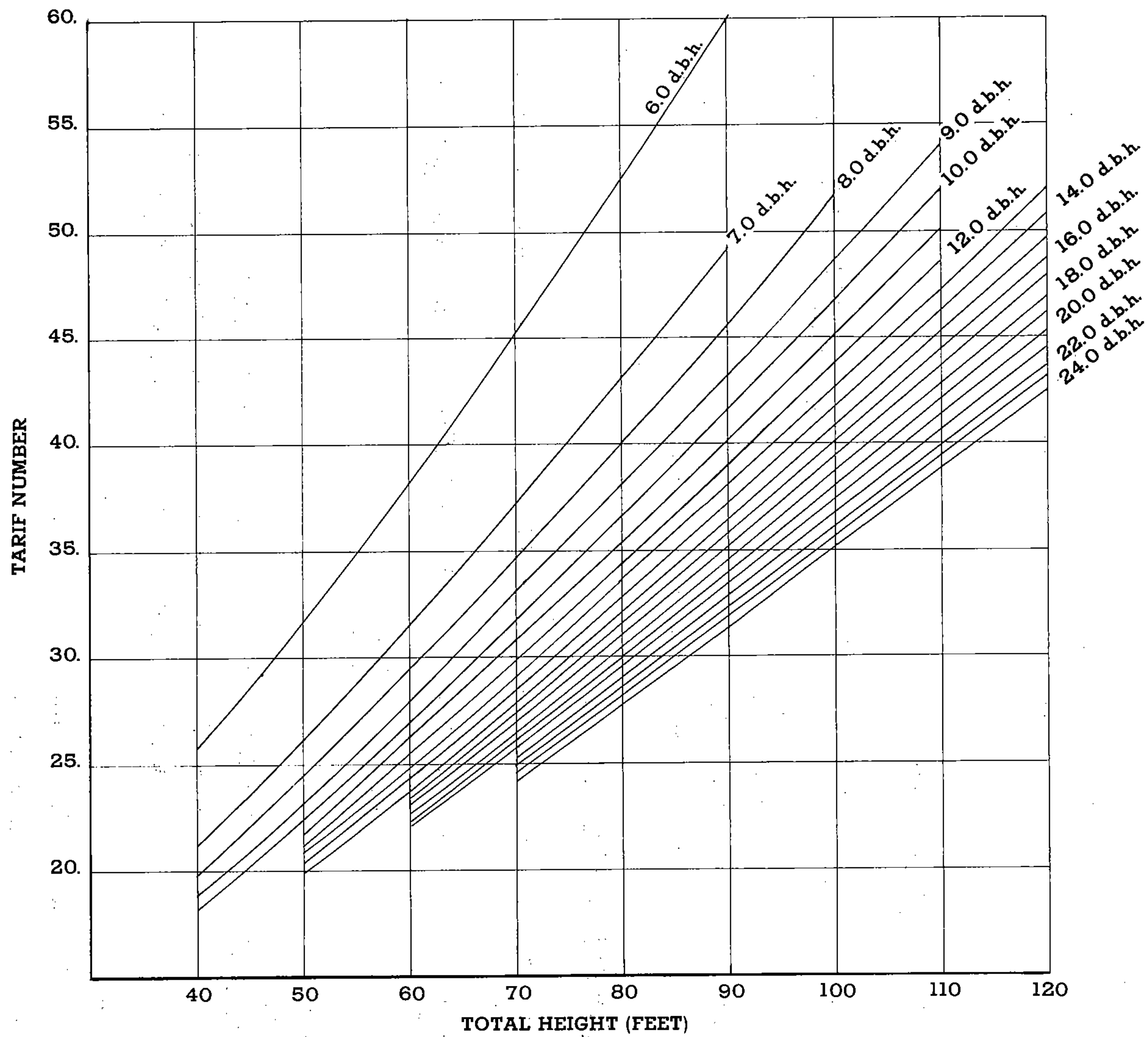

Figure 1. —Graph for estimating tarif number of red alder from d.b.h. and total height.

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2. The basic equation for total cubic volume above stump (CVT), derived from (1), is:

$$\text{CVT} = 0.00545415\ (\text{d.b.h.})^2(\text{H}-4.5)\ (\text{F})$$

where[3]

$$\begin{aligned}\text{F} = {} & 0.36510(\text{Z})^{2.5} - 7.9032(\text{Z})^{2.5}(\text{d.b.h.})(10^{-3}) + 3.2950(\text{Z})^{2.5}(\text{H})(10^{-3}) \\ & - 1.9856(\text{Z})^{2.5}(\text{H})(\text{d.b.h.})(10^{-5}) - 2.9668\ (\text{Z})^{2.5}(\text{H})^2(10^{-6}) \\ & + 1.5092(\text{Z})^{2.5}(\text{H})^{0.5}(10^{-3}) + 4.9395(\text{Z})^4(\text{d.b.h.})(10^{-3}) \\ & - 2.05937(\text{Z})^4(\text{H})(10^{-3}) + 1.5042(\text{Z})^{33}(\text{H})(\text{d.b.h.})(10^{-6}) \\ & - 1.1433(\text{Z})^{33}(\text{H})^{0.5}(10^{-4}) + 1.8090(\text{Z})^{41}(\text{H})^2(10^{-7})\end{aligned}$$

in which

$$\text{Z} = \frac{(\text{H} - \tfrac{1}{2} - (\text{d.b.h.})/24)}{(\text{H}-4.5)} = \frac{(\text{total height above stump in feet})}{(\text{total height above b.h. in feet})}$$

3. The estimates of total cubic volume (CVT) for trees of specified d.b.h. and height (H) given by equation 2 above may be converted to merchantable cubic volumes to top diameters of 4, 6, 8, 10, and 12 inches i.b. (CV4, CV6, CV8, CV10, CV12) by multiplying estimated total cubic volume by the following conversion factors:

$$\frac{\text{CV4}}{\text{CVT}} = 0.99875 - \frac{43.336}{(\text{d.b.h.})^3} - \frac{124.717}{(\text{d.b.h.})^4} + \frac{0.193437(\text{H})}{(\text{d.b.h.})^3} + \frac{479.83}{(\text{d.b.h.})^3(\text{H})}$$

$$\frac{\text{CV6}}{\text{CVT}} = 1.00081 - \frac{1614.44}{(\text{d.b.h.})^4} + \frac{2.86121(\text{H})}{(\text{d.b.h.})^4} + \frac{1686.7}{(\text{H})^3} - \frac{21.7181}{(\text{d.b.h.})(\text{H})} + \frac{1.1028(10^5)}{(\text{d.b.h.})^5(\text{H})}$$

$$\frac{\text{CV8}}{\text{CVT}} = 1.03361 - \frac{1.59234}{(\text{d.b.h.})} - \frac{4667.04}{(\text{d.b.h.})^4} + \frac{0.104498(\text{H})}{(\text{d.b.h.})^2} + \frac{5322.16}{(\text{d.b.h.})^3(\text{H})}$$

$$\frac{\text{CV10}}{\text{CVT}} = 1.02328 - \frac{43.4570}{(\text{d.b.h.})^2} - \frac{7626.29}{(\text{d.b.h.})^4} + \frac{76.7229(\text{H})}{(\text{d.b.h.})^4} - \frac{9954.576(\text{H})}{(\text{d.b.h.})^6} + \frac{116766.}{(\text{d.b.h.})^4(\text{H})}$$

$$\frac{\text{CV12}}{\text{CVT}} = 1.21396 - \frac{10.24325}{(\text{d.b.h.})} + \frac{6.54920}{(\text{H})} + \frac{12.2606(\text{H})}{(\text{d.b.h.})^3} - \frac{46116.8(\text{H})}{(\text{d.b.h.})^6} - \frac{1145.61}{(\text{d.b.h.})^2(\text{H})}$$

A similar set of conversion factors, some what less accurate but based on d.b.h. only, is given by the curves shown in figure 2.

These cubic-volume conversion factors, and the board-foot volume conversion factors of following sections, should of course *not* be applied to trees of dimensions too small to contain merchantable volume to the specified top diameter limit.

[3] "F" is $\int_0^{Z} \left[\frac{(\text{d.i.b.})}{(\text{d.b.h.})}\right]^2 dx$, which is a form factor based on a cylinder of diameter equal to d.b.h. and height equal to (H—4.5).

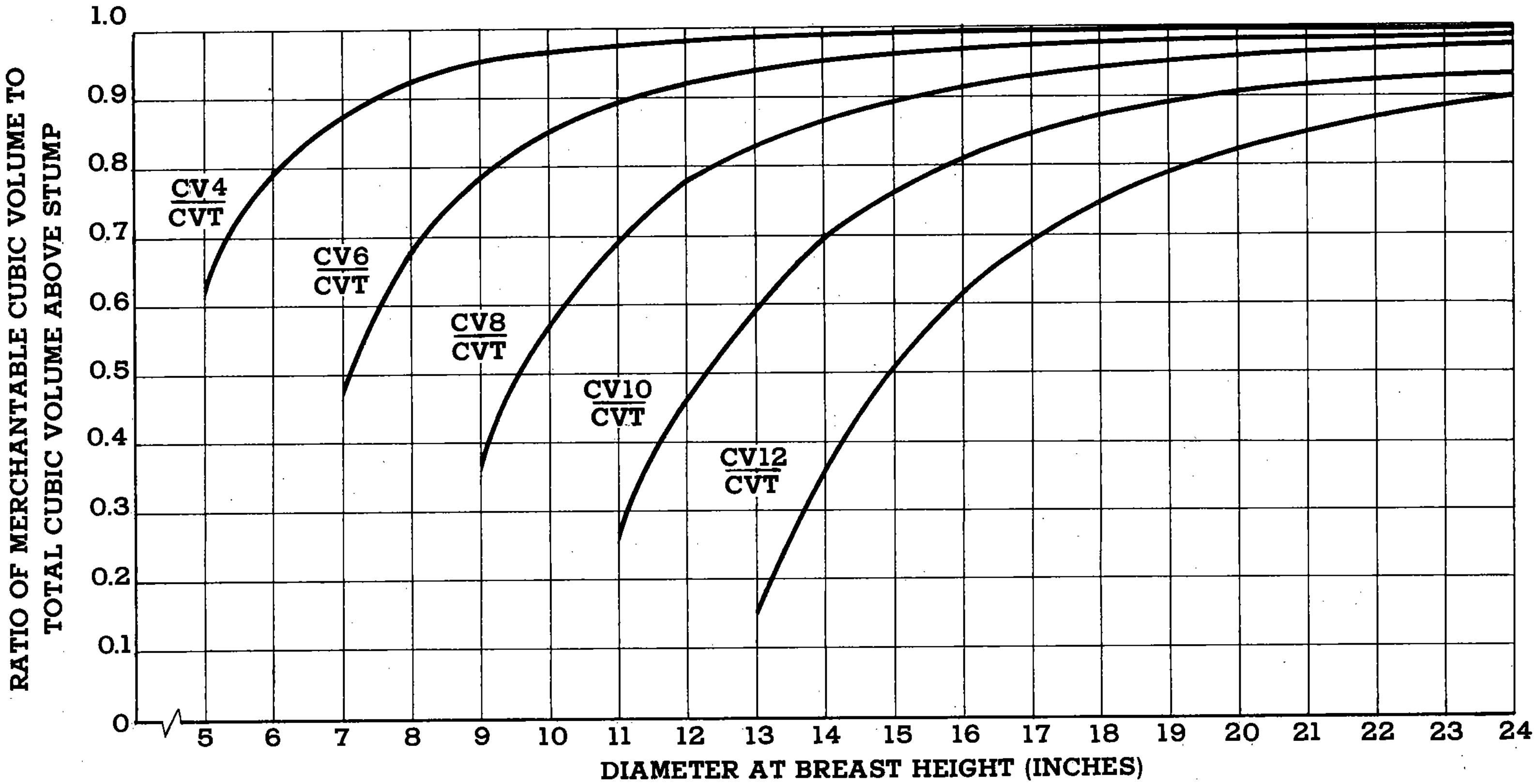

Figure 2.—Relationship of ratios of (merchantable cubic volume)/CVT to tree d.b.h.

4. Total cubic-volume estimates (CVT) for saw-log-size trees of specified d.b.h. and H may be converted to board feet, Scribner, by multiplying by the conversion factors given below. Ratios are for board-foot volumes to tops of 6, 8, 10, and 12 inches i.b. (SV6, SV8, SV10, and SV12). Scaling is assumed to be in 16-foot logs, with fractional diameters rounded down to next lower exact inch, since this most nearly approximates current commercial practice.

$$\frac{\text{SV6}}{\text{CVT}} = 8.36001 - \frac{30.3982}{\text{D}} - \frac{205.532}{\text{H}} - \frac{152.004}{\text{D}^2} + \frac{1674.00}{\text{D}^4} + \frac{1227.84}{\text{DH}}$$

$$\frac{\text{CV8}}{\text{CVT}} = 8.11342 - \frac{398.033}{\text{D}^2} - \frac{218.418}{\text{H}} + \frac{76702.1}{\text{H}^3} - \frac{0.05208(\text{H})}{\text{D}} + \frac{11337.8}{\text{D}^2\text{H}} - \frac{2.46759(10^7)}{\text{D}^6\text{H}}$$

$$\frac{\text{SV10}}{\text{CVT}} = 7.76152 - \frac{16.6331}{(\text{d.b.h.})} - \frac{156.070}{(\text{H})} - \frac{5845.33}{(\text{d.b.h.})^3} + \frac{2.29172(10^5)}{(\text{d.b.h.})^3(\text{H})} - \frac{2.43897(10^8)}{(\text{d.b.h.})^8}$$

$$\frac{\text{SV12}}{\text{CVT}} = 7.34627 - \frac{129.986}{(\text{H})} - \frac{2.20502(\text{H})(10^5)}{(\text{d.b.h.})^6} + \frac{7465.72}{(\text{H})^2} - \frac{4082.55}{(\text{d.b.h.})(\text{H})} + \frac{2.97332(10^6)}{(\text{d.b.h.})^4(\text{H})}$$

Total cubic-volume estimates (CVT) for saw-log-size trees of specified d.b.h. and H may be converted to board feet, International ¼-inch rule, by multiplying by the conversion factors given below. Ratios are for board-foot volumes to tops of 6, 8, 10, and 12 inches i.b. (IV6, IV8, IV10, and IV12). Scaling is assumed to be in 16-foot logs, with fractional diameters rounded to the nearest inch.

$$\frac{\text{IV6}}{\text{CVT}} = 8.22987 - \frac{189.164}{\text{D}^2} - \frac{5631.3}{\text{D}^4} - \frac{179.256}{\text{H}} + \frac{7977.6}{\text{D}^2\text{H}} + \frac{143.04(\text{H})}{\text{D}^5}$$

$$\frac{\text{IV8}}{\text{CVT}} = 8.30882 - \frac{190.146}{\text{D}^2} - \frac{21960.}{\text{D}^4} - \frac{191.413}{\text{H}} - \frac{1.211(10^6)}{\text{H}^4} + \frac{11281.}{\text{D}^2\text{H}}$$

$$\frac{\text{IV10}}{\text{CVT}} = 8.24035 - \frac{7034.29}{\text{D}^3} - \frac{4.22367(10^8)}{\text{D}^8} - \frac{178.007}{\text{H}} + \frac{2.49955(10^5)}{\text{D}^3\text{H}}$$

$$\frac{\text{IV12}}{\text{CVT}} = 7.84957 - \frac{51936.7}{\text{D}^2\text{H}} + \frac{398559.}{\text{D}^3\text{H}} - \frac{2.45274(10^5)(\text{H})}{\text{D}^6} - \frac{130.245}{\text{H}} + \frac{7.299(10^6)}{\text{H}^4}$$

Erratum: In the above equations D = d.b.h.

TABLE 1.--DISTRIBUTION OF SAMPLE TREES BY DIAMETER AND HEIGHT CLASSES.

DBH	TOTAL HEIGHT IN FEET											TOTAL FOR DBH CLASS
	20	30	40	50	60	70	80	90	100	110	120	
2	1	4	2	-	-	-	-	-	-	-	-	7
3	-	1	6	3	-	-	-	-	-	-	-	10
4	-	1	3	1	1	-	-	-	-	-	-	6
5	-	-	1	-	2	2	-	-	-	-	-	5
6	-	-	-	1	-	2	5	2	-	-	-	10
7	-	-	-	-	1	4	12	5	-	-	-	22
8	-	-	-	-	-	1	14	9	-	-	-	24
9	-	-	-	-	4	3	14	11	1	-	-	33
10	-	-	-	-	-	2	18	12	4	-	-	36
11	-	-	-	-	2	5	9	13	3	-	-	32
12	-	-	-	-	2	7	15	23	4	1	-	52
13	-	-	-	-	-	2	7	14	4	-	-	27
14	-	-	-	-	-	2	15	18	16	1	-	52
15	-	-	-	-	-	2	12	14	10	1	-	39
16	-	-	-	-	-	1	13	14	13	4	-	45
17	-	-	-	-	-	-	4	5	6	4	-	19
18	-	-	-	-	-	-	8	4	2	2	-	16
19	-	-	-	-	-	1	2	4	7	-	1	15
20	-	-	-	-	-	-	2	2	4	-	-	8
21	-	-	-	-	-	-	-	2	1	1	-	4
22	-	-	-	-	-	-	-	-	2	-	-	2
23	-	-	-	-	-	1	1	-	1	-	1	4
24	-	-	-	-	-	-	1	2	-	2	-	5
TOTAL FOR HT CLASS	1	6	12	5	12	35	152	154	78	16	2	TOTAL NO. 473